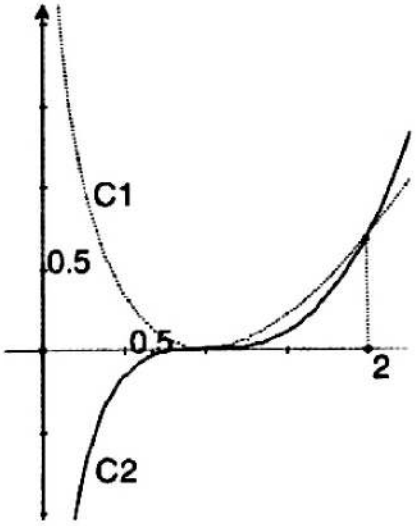

3) $\mathcal{A} = \int_1^2 (f_1(x) - f_2(x))\, dx$

$\mathcal{A} = \int_1^2 (-x^2 + 3x - 2).\, \mathrm{Ln}x\, dx$

Soit

$$\begin{cases} U'(x) = -x^2+3x-2 \\ V(x) = \mathrm{Ln}x \end{cases}$$

$$\Rightarrow \begin{cases} U(x) = \frac{-x^3}{3} + \frac{3x^2}{2} - 2x \\ V'(x) = 1/x \end{cases}$$

$$\mathcal{A} = \left[\left(\frac{-x^3}{3} + \frac{3x^2}{2} - 2x \right) \mathrm{Ln}x \right]_1^2 - \int_1^2 \left(\frac{-x^2}{3} + \frac{3x}{2} - 2 \right) dx$$

$$\mathcal{A} = -\frac{2}{3}\,\mathrm{Ln}2 - \left[\left(\frac{-x^3}{9} + \frac{3x^2}{4} - 2x \right) \right]_1^2$$

$$\mathcal{A} = \left(\frac{19}{36} - \frac{2}{3}\,\mathrm{Ln}2 \right) \quad \text{u.a}$$

B) $v_n = \sum_{k=0}^{n} \frac{(-1)^k}{k+1}$; $n \in \mathbb{N}$

1) $u_n = \int_1^2 f_n(x)\, dx$

Soit $u_n = \int_1^2 (x-1)^n \mathrm{Ln}x\, dx$

$$\begin{cases} (x-1)^n \xrightarrow{P} \frac{1}{n+1}(x-1)^{n+1} \\ \mathrm{Ln}x \xrightarrow{d} \frac{1}{x} \end{cases}$$

$$u_n = \left[\frac{(x-1)^{n+1}}{n+1} \mathrm{Ln}x \right]_1^2 - \frac{1}{n+1} \int_1^2 \frac{(x-1)^{n+1}}{x}\, dx$$

$$u_n = \frac{\mathrm{Ln}2}{n+1} - \frac{1}{n+1} \int_1^2 \frac{(x-1)^{n+1}}{x}\, dx$$

D'où

$$(n+1)\, u_n = \mathrm{Ln}2 - \int_1^2 \frac{(x-1)^{n+1}}{x}\, dx$$

2) pour $n \in \mathbb{N}^*$

a) $\mathrm{Ln}2 - (n+1)\, u_n = \int_1^2 \frac{(x-1)^{n+1}}{x}\, dx$

$1 \le x \le 2 \Rightarrow \frac{1}{2} \le \frac{1}{x} \le 1$

$$\Rightarrow \frac{1}{2}(x-1)^{n+1} \le \frac{(x-1)^{n-1}}{x} \le (x-1)^{n+1}$$

$$\Rightarrow \int_1^2 \frac{1}{2}(x-1)^{n+1}\, dx \le \int_1^2 \frac{(x-1)^{n+1}}{x}\, dx \le \int_1^2 (x-1)^{n+1}\, dx$$

Qarray.tn - Aqra Blech - Qarray.tn